Exercise 87

For the following exercises, for each polynomial, a. find the degree; b. find the zeros, if any; c. find the y-intercept(s), if any; d. use the leading coefficient to determine the graph's end behavior; and e. determine algebraically whether the polynomial is even, odd, or neither.

$$f(x) = 3x - x^3$$

Solution

Part (a)

The degree of the polynomial is 3 because the highest power of x is 3.

Part (b)

Set $f(x) = 0$.

$$f(x) = 3x - x^3 = 0$$

Factor the left side.

$$x(3 - x^2) = 0$$

$$x(\sqrt{3} + x)(\sqrt{3} - x) = 0$$

Therefore, the zeros are

$$x = \{-\sqrt{3}, 0, \sqrt{3}\}.$$

Part (c)

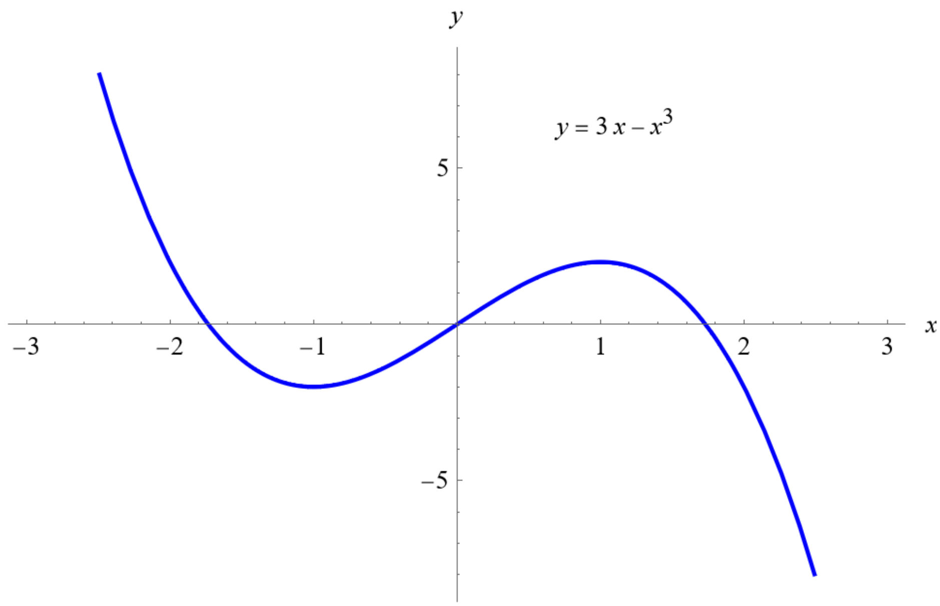
y-intercepts are the points where the function touches the y-axis, which occurs when $x = 0$.

$$f(0) = 3(0) - (0)^3 = 0$$

Therefore, there's one y-intercept: $(0, 0)$.

Part (d)

$-x^3$ is the dominant term in the polynomial, so the graph is cubic. Since the coefficient is -1, it goes up to the left and goes down to the right. The graph of $f(x)$ versus x below illustrates this.



Part (e)

Plug in $-x$ for x in the function.

$$\begin{aligned} f(-x) &= 3(-x) - (-x)^3 \\ &= -3x - (-x^3) \\ &= -3x + x^3 \\ &= -(3x - x^3) \\ &= -f(x) \end{aligned}$$

Since $f(-x) \neq f(x)$, the function $f(x)$ is not even.

Since $f(-x) = -f(x)$, the function $f(x)$ is odd.